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Investment project valuation based on a fuzzy binomial approach

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ABSTRACT

The typical approaches to project valuation are based on discounted cash flows (DCF) analysis which provides measures like net present value (NPV) and internal rate of return (IRR). DCF-based approaches exhibit two major pitfalls. One is that DCF parameters such as cash flows cannot be estimated precisely in an uncertain decision making environment. The other one is that the values of managerial flexibilities in investment projects cannot be exactly revealed through DCF analysis. Both of them would have significant influence on strategic investment projects valuation. This paper proposes a fuzzy binomial approach that can be used in project valuation under uncertainty. The proposed approach also reveals the value of flexibilities embedded in the project. Furthermore, this paper provides a method to compute the mean value of a project's fuzzy NPV. The project's fuzzy NPV is characterized with right-skewed possibilistic distribution because these flexibilities retain the upside potential of profit but limit the downside risk of loss. Finally, this paper discusses the value of multiple options in a project.

1. Introduction

DCF-based approaches to project valuation implicitly assume that a project will be undertaken immediately and operated continuously until the end of its expected useful life, even though the future is uncertain. For example, in the NPV approach we make implicit assumptions concerning a certain "expected scenario" of cash flows. By treating projects as independent investment opportunities, decisions are made to accept projects with positive computed NPVs. Since DCF-based approaches ignore the upside potentials of added value that could be brought to projects through managerial flexibilities and innovations, they usually underestimate the upside value of projects [4,11,17,26,30].

For high-risk investment projects, the traditional NPV method may adopt higher discount rates to discount project cash flows for trade-off or compensation. However, higher discount rates may result in the underestimation of project value and the rejection of a potential project. Investments such as new drug development or crude oil exploitation may carry high risk, but may also bring higher returns. In particular, as market conditions change in the future, investment project may include flexibilities by which project value can be raised. Such flexibilities are called real options or strategic options.

Real options analysis, as a strategic decision making tool, borrows ideas from financial options because it explicitly accounts for future flexibility value. Real options analysis is based on the assumption that there is an underlying source of uncertainty, such as the price of a commodity or the outcome of a research project. Over time, the outcome of the underlying uncertainty is revealed, and managers can adjust their strategy accordingly.

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In DCF, parameters such as cash flows and discount rates are difficult to estimate [6]. In particular, innovative investment projects may count on the subjective judgments of decision makers due to lack of past data for reference. These parameters are essentially estimated under uncertainty. With respect to uncertainty, probability is one way to depict whereas possibility is another. Fuzzy set theory provides a basis for the theory of possibility [32]. Fuzzy logic may be viewed as an attempt at formalization of two remarkable human capabilities. One is the capability to converse, reason and make rational decisions in an environment of imprecision, uncertainty and incompleteness of information and the other one is to perform a wide variety of physical and mental tasks without any measurements and computations [34]. The outstanding feature of fuzzy logic is that in fuzzy logic everything is—or is allowed to be—a matter of degree. In the generalized theory of uncertainty, uncertainty is linked to information through the concept of granular structure—a concept that plays a key role in human interaction with the real world [33]. Thus, these parameters can be characterized with possibilistic distributions instead of probabilistic distributions, and can be estimated by making use of fuzzy numbers.

The objectives of this paper are to develop a fuzzy binomial approach to evaluate a project embedded with real options, to propose a method suitable for computing the mean value of fuzzy NPV, and to explore the value of multiple options existing in projects. The paper is organized as follows. Section 2 provides a literature review of real options analysis. We especially focus on pricing, applications and recent developments of real options analysis. Section 3 presents a fuzzy binomial approach to evaluate a project under vague situations. This section also proposes a method to compute the mean value of fuzzy NPV. Section 4 illustrates a project valuation based on our approach. In the example, the premiums (or values) of the real options are also assessed. Section 5 discusses multiple options and limitations of the study. Finally, conclusions are drawn in Section 6.

2. Literature review

Traditional net present value techniques only focus on current predictable cash flows and ignore future managerial flexibilities, therefore, may undervalue the projects and mislead the decision makers. The real options approach to projects valuation seeks to correct the deficiencies of the traditional valuation methods through recognizing that managerial flexibilities can bring significant values to projects. According to real options theory, an investment is of higher value in a more uncertain or volatile market because of investment decision flexibilities.

Based on real options theory, Chen et al. [8] presented an approach to evaluate IT investments subject to multiple risks. By modeling public risks and private risks into a unified framework, they utilized the binomial model to evaluate an ERP development project. Wu et al. [29] argued that ERP may be best represented by a non-analytical, compound option model. However, most IT studies that employ the options theory only consider a single option, use an analytical model such as the Black and Scholes [1] model, and cannot deal with multi-option situations. Based on the real options theory, Wu et al. employed the binomial tree approach to implement an active ERP management which involves uncertainties over time.

Hahn and Dyer [13] proposed a recombining binomial lattice approach for modeling real options and valuing managerial flexibility to address a common issue in many practical applications—underlying stochastic processes that are mean-reverting. The models were tested by implementing the lattice in binomial decision tree format and applying to a real application by solving for the value of an oil and gas switching option.

Reyck et al. [21] proposed an alternative approach for valuing real options based on the certainty-equivalent version of the NPV formula, which eliminates the need to identify market-priced twin securities. Moreover, Bowe and Lee [3] also utilized the log-transformed binomial lattice approach to evaluate the Taiwan High-Speed Rail (THSR) project.

Basically, if the values of parameters in a valuation model are numeric, they come from the probabilistic expected values of these parameters. However, the values of parameters can also be estimated as fuzzy numbers to characterize the uncertainty in terms of possibility rather than probability.

By modeling the stock price in each state as a fuzzy number, Muzzioli and Torricelli [20] obtained a possibility distribution of the risk-neutral probability in a multi-period binomial model, then computed the option price with a weighted expected value interval, and thus determined a "most likely" option value within the interval. Muzzioli and Reynaerts [19] also addressed that the key input of the multi-period binomial model is the volatility of the underlying asset, but it is an unobservable parameter. The volatility parameter can be estimated either from historical data (historical volatility) or implied from the price of European options (implied volatility). Providing a precise volatility estimate is difficult; therefore, they used a possibility distribution to model volatility uncertainty and to price an American option in a multi-period binomial model.

Carlsson and Fuller [6] mentioned that the imprecision in judging or estimating future cash flows is not stochastic in nature, and that the use of the probability theory leads to a misleading level of precision. Their study introduced a (heuristic) real option rule in a fuzzy setting in which the present values of expected cash flows and expected costs are estimated by trapezoidal fuzzy numbers. They determined the optimal exercise time with the help of possibilistic mean value and variance of fuzzy numbers. The proposed model that incorporates subjective judgments and statistical uncertainties may give investors a better understanding of the problem when making investment decisions. Carlsson et al. [7] also developed a methodology for valuing options on R& D projects, in which future cash flows were estimated by trapezoidal fuzzy numbers. In particular, they presented a fuzzy mixed integer programming model for the R& D optimal portfolio selection problem.

In addition to the binomial model, the Black–Scholes model is another way to evaluate the option's value. Owing to fluctuations in the financial market from time to time, some input parameters in the Black–Scholes formula cannot be expected to always be precise. Wu [28] applied the fuzzy set theory to the Black–Scholes formula. Under the assumptions of fuzzy

interest rate, fuzzy volatility and fuzzy stock price, the European option price turns into a fuzzy number. This allows the financial analyst to pick a European option price with an acceptable degree of belief.

Lee et al. [15] adopted the fuzzy decision theory and Bayes' rule as a basis for measuring fuzziness in the practice of option analysis. Their study also employed "Fuzzy Decision Space" that consisted of four dimensions—fuzzy state, fuzzy sample information, fuzzy action and evaluation function—to describe the decisions of investors. These dimensions were used to derive a fuzzy Black–Scholes option pricing model under fuzzy environments.

Thiagarajah et al. [25] also addressed that most stochastic models involve uncertainty arising mainly from lack of knowledge or from inherent vagueness. Traditionally, these stochastic models are solved using probability theory and fuzzy set theory. In their study, using adaptive fuzzy numbers, they modeled the uncertainty of characteristics such as interest rate, volatility, and stock price. They also replaced fuzzy interest rate, fuzzy stock price and fuzzy volatility with possibilistic mean values in the fuzzy Black–Scholes formula.

Making a R&D portfolio decision is difficult, because the long lead times of R&D and the market and technology dynamics lead to unavailable or unreliable collected data for portfolio management. Wang and Hwang [27] developed a fuzzy R&D portfolio selection model to hedge against the R&D uncertainty. Since traditional project valuation methods often underestimated the risky project, a fuzzy compound-options model was used to evaluate the value of each R&D project. The R&D portfolio selection problem was formulated as a fuzzy zero-one integer programming model that could handle both uncertain and flexible parameters to determine the optimal project portfolio.

From the viewpoint of fuzzy random variables, Yoshida et al. [31] discussed, under uncertainty in financial engineering, an American put option model that was based on the Black–Scholes stochastic model. In their study, probability is applied as the uncertainty such that something occurs or not with probability, and fuzziness is applied as the uncertainty such that the exact values cannot be specified because of a lack of knowledge regarding the present stock market. By introducing fuzzy logic to the log-normal stochastic processes for the financial market, they presented a model with uncertainty of both randomness and fuzziness in output.

The Garman–Kohlhagen (G–K) model is a closed-form solution of the European currency options pricing model based on the Black–Scholes model, but the input variables of the G–K model are usually regarded as real numbers. However, it is more suitable and realistic to price currency options with fuzzy numbers because these variables are only available with imprecise data or data related in a vague way. Therefore, Liu [16] started from the fuzzy environments of currency options markets, introduced fuzzy techniques, and created a fuzzy currency options pricing model. By turning exchange rate, interest rates and volatility into triangular fuzzy numbers, the currency option price turns into a fuzzy number. This allows financial investors to pick any currency option price with an acceptable degree of belief.

Combining real options theory with other theories has been a significant development in recent years. Smit and Trigeorgis [23,24] combined real options theory with game theory to serve as an analytical instrument for competitive strategies in an uncertain environment. They unify two major strands of economic theory—real options and games—into a single, coherent framework and demonstrate how these ideas can be applied to formulating corporate strategy. The integrated options-and-games perspective is particularly relevant for oligopolistic and innovative industries such as consumer electronics, telecommunications or pharmaceuticals.

From a modeling perspective, real options valuation methods have tended to follow financial option pricing techniques. The Black–Scholes models are used to evaluate simple real option scenarios such as delay decisions, research and development, licenses, patents, growth opportunities, and abandonment scenarios [18]. Despite its theoretical appeal, however, the practical use of real option valuation techniques in industry has been limited by the complexity of these techniques, the resulting lack of intuition associated with the solution process, or the restrictive assumptions required for obtaining analytical solutions.

On the other hand, Cox et al. [10] developed a binomial discrete-time option valuation technique that has gained similar popularity to evaluate real options due to its intuitive nature, ease of implementation, and wide applicability to variety of option attributes. In addition, analytical models such as the Black–Scholes formula focus on a single option and cannot deal with multi-option situations. Therefore, we adopt the binomial model as a basis to develop the fuzzy valuation approach to projects valuation.

3. The valuation approach

3.1. Expanded net present value

The NPV approach assumes a fixed scenario, in which a company starts and completes a project that then generates cash flows during some expected lifetime without any contingencies. The approach anticipates no contingency for delaying or abandoning the project if market conditions turn sour. However, the assumption about NPV does not fit the actual situation. In reality, if the market is unfavorable, the project could be postponed to undertake until market conditions turn better; or, the project may be abandoned during the operation to reduce losses; or, the project may be expanded or extended as market conditions turn around. The flexibilities of these investment decisions indicate that decision makers are capable of restricting loss risks and retaining the potential to raise profits infinitely. As a result, the valuation should include these flexibilities which are embedded as real options in investment projects.

In considering option value, the traditional NPV can be expanded as: expanded NPV = static NPV + value of option from active management [26]. The expanded NPV is also called strategic NPV. Static NPV is the NPV obtained using the traditional discount method; it is also called passive NPV.

3.2. The fuzzy binomial valuation approach

In this study, a fuzzy binomial valuation approach is proposed to evaluate investment projects that are embedded with real options. The value of the project is represented by its expanded NPV, which can be evaluated by the valuation approach. However, the parameters are estimated by fuzzy numbers when the expanded NPV is estimated; thus, the expanded NPV is called fuzzy expanded NPV (*FENPV*) in this study.

The proposed valuation approach is based on Cox et al. [10]. Assuming there is a call option with the present value of underlying asset S_0 and exercising price K, the value of the underlying asset has P_u probability to rise to uS_0 or P_d probability to drop to dS_0 in the next period. The factors u and d represent the jumping up and down factors of the underlying asset's present value, respectively. A single period binomial tree of the underlying asset value is shown in Fig. 1.

The option will be exercised at period $t = 1$ if the underlying value is higher than K, and forgone if the underlying value is lower than K. The dynamics of the option value is shown in Fig. 2.

If the option is sold at price C_0, then the pricing approach is generally based on the assumption of replicating portfolio and can thus be determined by the following expression:

$$C_0 = \frac{1}{(1+r)}[P_u C_{1u} + P_d C_{1d}] \tag{1}$$

in which r is risk-free interest rate, and P_u and P_d are risk-neutral probabilities, which are determined by the following formulas.

$$P_u = \frac{(1+r)-d}{(u-d)}, \tag{2}$$

$$P_d = \frac{u-(1+r)}{(u-d)} = 1 - P_u. \tag{3}$$

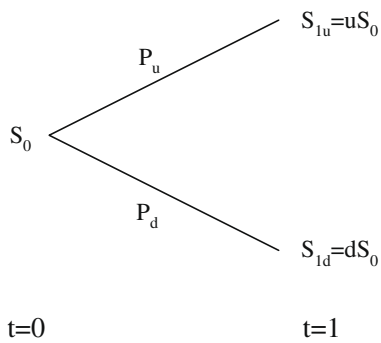

Fig. 1. The single period binomial tree of underlying asset value.

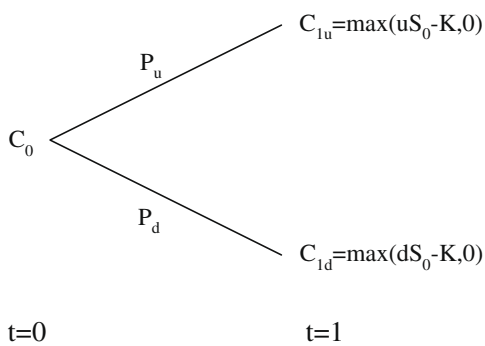

Fig. 2. The dynamics of option value.

Therefore, the price or present value of the call option is the discounted result of the option values C_{1u} and C_{1d} with risk-neutral probabilities. Also, under the assumption of no arbitrage opportunities, the condition $0 < d < 1 < (1 + r) < u$ must be satisfied. Furthermore, the expected return of the underlying asset should be zero based on the no-arbitrage assumption:

$$P_u\left(\frac{uS_0}{1+r} - S_0\right) + P_d\left(\frac{dS_0}{1+r} - S_0\right) = 0. \tag{4}$$

That is

$$\frac{uP_u}{1+r} + \frac{dP_d}{1+r} = 1. \tag{5}$$

Thus, we have the following risk-neutral probabilities equations:

$$\begin{cases} P_u + P_d = 1 \\ \frac{uP_u}{1+r} + \frac{dP_d}{1+r} = 1 \end{cases}. \tag{6}$$

From (1)–(3), we know that the main factors affecting the call option value are jumping factors u and d; it is not easy, however, to estimate their values in a precise manner due to the uncertainty of the underlying volatility.

The cash flow models applied to many financial decision making problems often involve some degree of uncertainty. In the case of deficient data, most decision makers tend to rely on experts' knowledge of financial information when carrying out their financial modeling activities. The nature of this knowledge often tends to be vague rather than random. Fuzzy theory, which is aimed at rationalizing the uncertainty caused by vagueness or imprecision, has provided a promising basis for manipulating such vague knowledge [22]. In the relevant application of financial decision making, there is an example of employing triangular fuzzy numbers to examine the profitability indexes [9].

In strategic or innovative investment projects, information often tends to be vague rather than random. Therefore, this study considers possibilistic uncertainty rather than probabilistic uncertainty and employs fuzzy numbers instead of statistics to estimate the parameters. For lightening computation efforts, we utilize the triangular fuzzy numbers $\tilde{u} = [u1, u2, u3]$ and $\tilde{d} = [d1, d2, d3]$ to represent the jumping factors of the underlying asset. Therefore, the risk-neutral probabilities equations can be rewritten as

$$\begin{cases} \tilde{P}_u \oplus \tilde{P}d = \tilde{1} \\ \frac{\tilde{u}\otimes\tilde{P}_u}{1+r} \oplus \frac{\tilde{d}\otimes\tilde{P}d}{1+r} = \tilde{1} \end{cases}, \tag{7}$$

where $\tilde{P}_u = [P_{u1}, P_{u2}, P_{u3}]$ and $\tilde{P}d = [P_{d1}, P_{d2}, P_{d3}]$. Thus, we have

$$\begin{cases} [P_{u1}, P_{u2}, P_{u3}] \oplus [P_{d1}, P_{d2}, P_{d3}] = [1, 1, 1] \\ \frac{[u1,u2,u3]\otimes[P_{u1},P_{u2},P_{u3}]u}{1+r} \oplus \frac{[d1,d2,d3]\otimes[P_{d1},P_{d2},P_{d3}]}{1+r} = [1, 1, 1] \end{cases}, \tag{8}$$

which are

$$\begin{cases} P_{ui} + P_{di} = 1 \\ \frac{ui\times P_{ui}}{1+r} + \frac{d_i\times P_{di}}{1+r} = 1 \end{cases} \quad \text{for } i = 1, 2, 3. \tag{9}$$

And can be solved as

$$P_{ui} = \frac{(1+r) - d_i}{u_i - d_i}, \tag{10}$$

$$P_{di} = \frac{u_i - (1+r)}{u_i - d_i}. \tag{11}$$

Since the risk-free interest rate r and the exercising price K are usually known, they are crisp values, whereas, the option values C_{1u} and C_{1d} become fuzzy numbers as a result of the jumping factors being fuzzified. That is, $\tilde{C}_{1u} = \max(\tilde{u}S_0 - K, 0)$ and $\tilde{C}_{1d} = \max(\tilde{d}S_0 - K, 0)$. The ranking of two triangular fuzzy numbers $\tilde{A} = [a1, a2, a3]$ and $\tilde{B} = [b1, b2, b3]$ can be derived from $\max(\tilde{A}, \tilde{B}) = [\max(a1, b1), \max(a2, b2), \max(a3, b3)]$. Thus, the pricing formula for the fuzzy call option is

$$\tilde{C}_0 = \frac{1}{1+r}[\tilde{P}d \otimes \tilde{C}_{1d} \oplus \tilde{P}_u \otimes \tilde{C}_{1u}]. \tag{12}$$

In practical application, the present value of the underlying asset is determined by the NPV of the investment project; the exercising price is the additional outlay to exercise the embedded option.

Managerial flexibility to adopt future actions introduces an asymmetry or skewness in the probability distribution of the project NPV [30]. In the absence of such managerial flexibility, the probability distribution of project NPV would be considerably symmetric. However, in the existence of managerial flexibility such as the exercising of options, enhanced upside potential is introduced and the resulting actual distribution is skewed to the right.

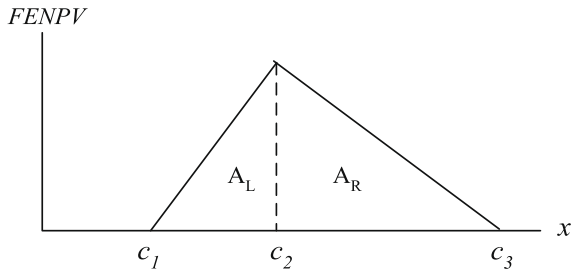

Fig. 3. A *FENPV* with right-skewed distribution.

In essence, identical results are obtained in the case of possibilistic distribution which is adopted by this study to characterize the NPV of an investment project. In other words, the characteristic of right-skewed distribution also appears in the *FENPV* of an investment project when the parameters (such as cash flows) are characterized with fuzzy numbers. Although many studies have proposed a variety of methods to compute the mean value [5,12] and median value [2] of fuzzy numbers, these works did not consider the right-skewed characteristic present in the *FENPV*. Therefore, this study proposes a new method to compute the mean value of the *FENPV* based on its right-skewed characteristic. This mean value can be used to represent the *FENPV* with a crisp value. Moreover, different *FENPV*s can be compared according to their mean values.

Let $\tilde{C} = [c_1(\alpha), c_3(\alpha)]$ be a fuzzy number and $\lambda \in [0,1]$. Then, the mean value of $\tilde{C}$ is defined as

$$E(\tilde{C}) = \int_0^1 [(1-\lambda)c_1(\alpha) + \lambda c_3(\alpha)]d\alpha. \tag{13}$$

The weighted index λ is called the pessimistic–optimistic index in [31], but the index is determined by a subjective decision in [31]. However, this study considers that the index can be determined objectively. Fig. 3 illustrates a case in which the *FENPV* is represented by a right-skewed triangular fuzzy number. The right-skewed characteristic of *FENPV*—meaning that the more skew to the right, the more optimistic the payoff of the project—provides a clue to determining λ with $\lambda = \frac{AR}{AL+AR}$, where A_L and A_R are the left-part area and right-part area of the *FENPV*, respectively. Thus, when λ is determined objectively and substituted into (13), the mean value of the *FENPV* can be computed as follows:

$$E(FENPV) = \frac{(1-\lambda)c_1 + c_2 + \lambda c_3}{2}. \tag{14}$$

4. Illustrated example

This section employs the proposed valuation approach to assess a BOT (i.e. build, operate and transfer) transportation project. The BOT structure is an approach developed recently in order to encourage private sector participation in public infrastructure projects. Under the BOT model, the private sector undertakes to finance, design, build, operate and manage the infrastructure facility, and then transfer the asset, free of charge, to the host government following a specified concession period.

In the Taipei urban area, a mass transportation system has been chosen to be implemented using the BOT model, with a 10-year concession period after the system has been implemented. The transportation system will have 10 stations on a 48.6 km route. The project requires NT$ 88 billion to be invested before operation in order to implement the civil works construction, the master engineering workshop and infrastructure maintenance facilities. According to experts' forecasting, the mean value of passenger demand is 3025 million km per year, the standard deviation is 544.37 million km. Thus, the coefficient of variation (C.V.) is 0.18. In the operation stage, the ticket price is NT$ 8, the cost of operating and financing is NT$ 9600 million per year and the revenue is NT$ 24,200 (3025×8) million per year. The main factors affecting the operating revenue are ticket price and passenger demand. The ticket price, however, is supervised by the government and is fixed at NT$ 8 per km during the operation period. Therefore, passenger demand is the only factor that will impact the operating revenue.

In order to obtain a reasonable estimation of the present value of expected operating cash flows, an appropriate weighted average cost of capital (WACC) of the project needs to be identified in advance. This estimate also forms part of the required input for calculating the risk-adjusted rate used to discount cash flows for the NPV analysis. Assuming that the annual return on equity (ROE) is 18%, the annual tax rate is 25%, the total construction cost combines 70% project financing with 30% private equity capital, and the average of the capital-loan interest rates is 8.75%, the WACC of the project is estimated as follows

$$\text{WACC} = 0.18 \times 0.3 + 0.0875 \times (1 - 0.25) \times 0.7 = 0.1.$$

Using this WACC of 10% as the discount rate, the NPV of the project is

$$\mathrm{NPV} = \sum_{t=1}^{10} \frac{24,200}{(1+0.1)^t} - \sum_{t=1}^{10} \frac{9600}{(1+0.1)^t} - 88,000 = 1712 \quad (\text{million}).$$

As mentioned earlier, future passenger demand will affect the operating revenue, and will accordingly change the value of the project. Therefore, future passenger demand is a source of uncertainty that will impact the value of the project. However, the project has the flexibility to alter its scale of operation in the fourth year. That is, in the fourth year, the project can be expanded if passenger demand is rising or can be shrunk if passenger demand is falling. This flexibility takes the form of enabling the project to either expand its scale of operation by 45% on incurring an additional investment outlay I_E = NT$ 1 billion or to reduce its scale of operation by 45% and save a future operating cost I_C = NT$ 1 billion. In other words, the project has an embedded option to expand or shrink its scale of operation. We will evaluate this project with both embedded options, that is, with the option to either expand its scale or to contract its scale, respectively; we will then evaluate this project when the two options exist simultaneously.

Because the value of flexibility stems from the uncertainty of future passenger demand, we have to forecast future passenger demand. Here, the uncertainty is characterized as possibility instead of probability. Thus, a triangular fuzzy number is adopted to represent the C.V. of forecasting passenger demand, which stands for the volatility of passenger demand. According to experts' estimations, the C.V. might have a variation of ±20%. Hence, the triangular fuzzy number $\tilde{\rho} = [(1-0.2)\times 0.18, 0.18, (1+0.2)\times 0.18] = [0.144, 0.18, 0.216]$ is used to represent the volatility of passenger demand.

Based on the fuzzy volatility $\tilde{\rho}$, the fuzzy jumping factors $\tilde{u}$ and $\tilde{d}$ can be determined as $\tilde{u} = \exp(\tilde{\rho} \otimes \sqrt{\tau})$ and $\tilde{d} = 1/\tilde{u}$, where τ is the chosen time interval size expressed in the same unit as $\tilde{\rho}$ and exp denotes the exponential function. In this case, the value of τ is 1 because the volatility is estimated annually. As a result, we have $\tilde{u} = [1.1549, 1.1972, 1.2411]$ and $\tilde{d} = [0.8057, 0.8353, 0.8659]$. According to NPV = 1.712 billion, a binomial tree is established and shown in Fig. 4. (For simplicity, the numbers in the binomial tree are represented to two digits after the decimal point.) The binomial tree reveals that project value fluctuates with passenger demand.

The decision nodes in the fourth year involve the decision to exercise an option. The decision of whether to undertake the option to expand in the fourth year depends on whether or not market conditions are favorable at the time. The option to expand will be undertaken if market conditions are favorable, and will be foregone if market conditions are unfavorable; thus, the decision making will be $\max(\tilde{V}, 1.45\tilde{V} - I_E)$ at the time. A decision tree could be established to determine the *FENPV* of the project, which has an embedded option to expand its scale of operation (see Fig. 5). For example, the value at the topmost node of the third year is computed as follows:

$$[\tilde{P}_u \otimes \max(\tilde{V}^+, 1.45\tilde{V}^+ - I_E) \oplus \tilde{P}d \otimes \max(\tilde{V}^-, 1.45\tilde{V}^- - I_E)]/(1+r) = [2.20, 3.31, 4.77],$$

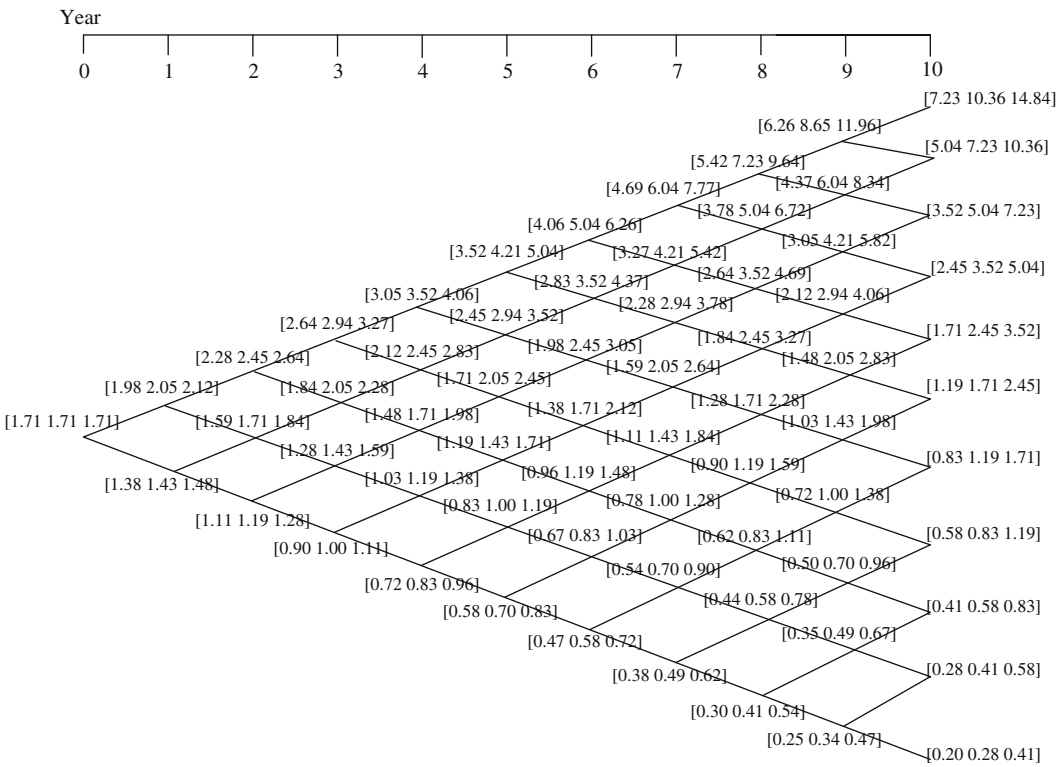

Fig. 4. Binomial tree of project value.

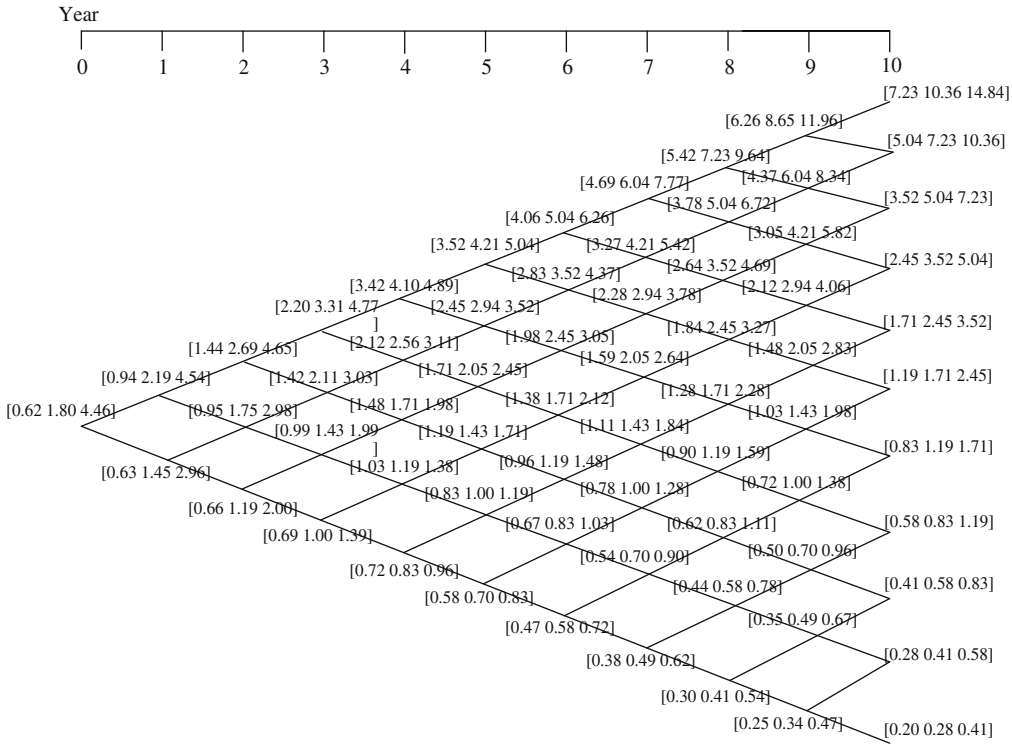

Fig. 5. The decision tree with the option to expand.

where I_E = 1 billion, $\tilde{P}_u = [0.49, 0.59, 0.70]$, $\tilde{P}d = [0.30, 0.41, 0.51]$, and r = 0.05. The root value of the decision tree, [0.62, 1.80, 4.46] (billion), is the *FENPV* of the project. Since the possibilistic distribution of this *FENPV* is right-skewed and the mean value of this *FENPV* is 2.54 (billion), the premium of the option to expand is 2.54–1.71 = 0.83 (billion).

In contrast to the above scenario, if the market conditions are unfavorable, the project has the option to reduce its scale of operation by *c%* (= 45%) in the fourth year for the purpose of saving future operation costs I_C (= NT$ 1 billion). In this case, the decision making in the fourth year will become $\max(\tilde{V}, 0.55\tilde{V} + I_C)$. Meanwhile, the *FENPV* of this project is [0.67, 1.85, 4.35] (billion), the mean value of this *FENPV* is 2.51 (billion), and the premium of this option to contract is 0.80 (billion).

Finally, in practical applications, the options may be embedded not only with individual but also with multiple at a specific time. This means that the decision makers have multiple choices to select, according to the market conditions at the time. We now consider a situation in which the option to expand and to contract exist simultaneously in the project. In this situation, the decision making is in the form of $\max(\tilde{V}, 1.45\tilde{V} - I_E, 0.55\tilde{V} + I_C)$, and the *FENPV* of the project increases to [0.69, 1.94, 4.68] (billion). Hence, the mean value of this *FENPV* and the premium of the multiple options increase to 2.68 and 0.97, respectively. The valuation results are summarized in Table 1.

The results in Table 1 show that the project value (assessed by the project *FENPV* and its mean value) has been raised by different embedded options. The project with multiple options possesses not only the highest project value but also the highest option premium. Yet, the premium of multiple options does not equate directly to the addition of the premiums of the option to expand and the option to contract. The premium cannot be raised linearly because of the nonlinear operations in the valuation model. Moreover, the mean values of the *FENPV*s are higher than the original project value of NT$ 1.712 billion. This reveals that the value of the project with embedded real options is higher than the value of the project without options and implies that the managerial flexibilities in a project have specific values that should not be ignored in the project valuation process.

Table 1
A summary of the results (in billion NT$).

Type of option	*FENPV* of the project	Mean value of the *FENPV*	Option premium
Expand	[0.62, 1.80, 4.46]	2.54	0.83
Contract	[0.67, 1.85, 4.35]	2.51	0.80
Multiple	[0.69, 1.94, 4.68]	2.68	0.97

5. Discussion

Real options may exist in different forms such as deferring, abandoning, contracting, expanding, extending or switching. In the previous example, we evaluated a project not only with a single option but also with multiple options. According to the results, the option to expand and the option to contract have approximate premiums for the project; however, the multiple options that contain these two options simultaneously have a higher premium for the project. This is a reasonable result because multiple options bring more flexibilities than single options. This does not mean, however, that different options can always be combined into multiple options. For example, logically, the option to defer an investment cannot simultaneously exist with the option to extend the investment. Furthermore, if the multiple options are undertaken at different time periods or of the various options are undertaken sequentially, they will entail different values for the project.

The single source of uncertainty is a limitation of this study. In this study, we consider just one source of uncertainty (i.e. passenger demand) for the project although multiple uncertainties may occur in practical cases. For example, Chen et al. [8] have proposed a real options valuation model to evaluate an information technology project with multiple risks.

6. Conclusions

Although options-related commodities have existed in the financial market for some time, they were not widely accepted because the prices of the commodities were based on the subjective judgments of decision makers. In 1973, Black–Scholes proposed a valuation model that allowed investors to price the options; investors no longer needed to rely on subjective judgments. Since then, transactions and innovations in options have continually developed. Options have become the most popular financial commodities and have satisfied the market's needs for hedge and arbitrage.

Similarly, conventional capital budgeting methods cannot discover the value of managerial flexibilities or other alternatives that exist in investment projects because estimating the value of flexibilities is difficult. As a result, a potential investment project will be undervalued and rejected. The rejection of a potential project may incur substantial losses. Nevertheless, once flexibilities are regarded as real options, they can be evaluated using option valuation models. The values of these flexibilities become measurable and the entire value of an investment project can be revealed.

The binomial valuation technique has gained popularity in the valuation of real options due to its intuitive nature, ease of implementation, and capability of dealing with multiple options. Furthermore, in an uncertain economic decision making environment, information such as cash flows, interest rate, cost of capital, and so forth possess some vagueness but not randomness [14]. Consequently, this study has proposed the fuzzy binomial valuation approach to evaluate investment projects with embedded real options in uncertain decision making environments and has applied this approach to practically evaluate an investment project.

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